

# DWF Calculation of Nucleon $G_A/G_V$ \*

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$G_A$  and  $G_V$  of nucleon as additional tests of domain wall fermion in baryon sector, with

- quenched lattice ( $16^3 \times 32$  at  $\beta = 6.0$ ) with DWF ( $L_s = 16$  and  $M_5 = 1.8$ ),
- following up a success in  $J^P = (1/2)^- N^*(1535)$  description,
- tensor charge is obtained as a byproduct.

From neutron  $\beta$  decay, we know  $G_V = G \cos \theta_c$  and  $G_A/G_V = 1.2670(35)$ :

- $G_V = G \lim_{q^2 \rightarrow 0} g_V(q^2)$  with  $\langle n | V_\mu^-(x) | p \rangle = i\bar{u}_n [\gamma_\mu g_V(q^2) + q_\lambda \sigma_{\lambda\mu} g_M(q^2)] u_p e^{-ipx}$ ,
- $G_A = G \lim_{q^2 \rightarrow 0} g_A(q^2)$  with  $\langle n | A_\mu^-(x) | p \rangle = i\bar{u}_n \gamma_5 [\gamma_\mu g_A(q^2) + q_\mu g_P(q^2)] u_p e^{-ipx}$ .

DWF makes  $G_A/G_V$  particularly easy:

- $Z_A = Z_V$ , so that  $g_A^{\text{lattice}}/g_V^{\text{lattice}}$  directly yields the continuum value.

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Kaplan's DWF<sup>1</sup>:  $D_5 = \gamma_\mu \partial_\mu + \gamma_5 \partial_s + M(s)$ , with

- $M(0) = 0$  and monotonic,

has a 4D zero-mode pinned at  $s = 0$ :  $\Psi(x, s) = \exp(ip_\mu x_m u) \phi(s) u_\pm$ ,

- $\gamma_5 u_\pm = \pm u_\pm$ ,
- $[\pm \partial_s + M(s)]\phi(s) = 0$ , or

$$\phi(s) = \exp[\mp \int_0^s M(s') ds'].$$

Shamir's<sup>2</sup>: 5D  $L^3 \times N_t \times L_s$  lattice,  $x \in L^3 \times N_t$  and  $s = 0, 1, , \dots, L_s - 1$ ,

- 5D fermion  $\Psi(x, s)$  with  $D_{x,s;x',s'} = \delta_{s,s'} D_{x,x'}^\parallel + \delta_{x,x'} D_{s,s'}^\perp$  defined by

$$D_{x,x'}^\parallel = \frac{1}{2} \sum_{\mu=1}^4 [(1 - \gamma_\mu) U_{x,\mu} \delta_{x+\hat{\mu},x'} + (1 + \gamma_\mu) U_{x',\mu}^\dagger \delta_{x-\hat{\mu},x'}] + (M_5 - 4) \delta_{x,x'},$$

$$D_{s,s'}^\perp = \frac{1}{2} [(1 - \gamma_5) \delta_{s+1,s'} + (1 + \gamma_5) \delta_{s-1,s'} - 2 \delta_{s,s'}] - \frac{m_f}{2} [(1 - \gamma_5) \delta_{s,L_s-1} \delta_{0,s'} + (1 + \gamma_5) \delta_{s,0} \delta_{L_s-1,s'}],$$

- $M_5$  represents domain walls, providing

- left-handed (at  $s = 0$ ) and right-handed (at  $s = L_s - 1$ ) light modes,
- coupled by  $m_f$ .

- 4D quark is defined through projections  $P_{R,L} = (1 \pm \gamma_5)/2$ ,

$$q(x) = P_L \Psi(x, 0) + P_R \Psi(x, L_s - 1) \text{ and } \bar{q}(x) = \bar{\Psi}(x, L_s - 1) P_L + \bar{\Psi}(x, 0) P_R$$

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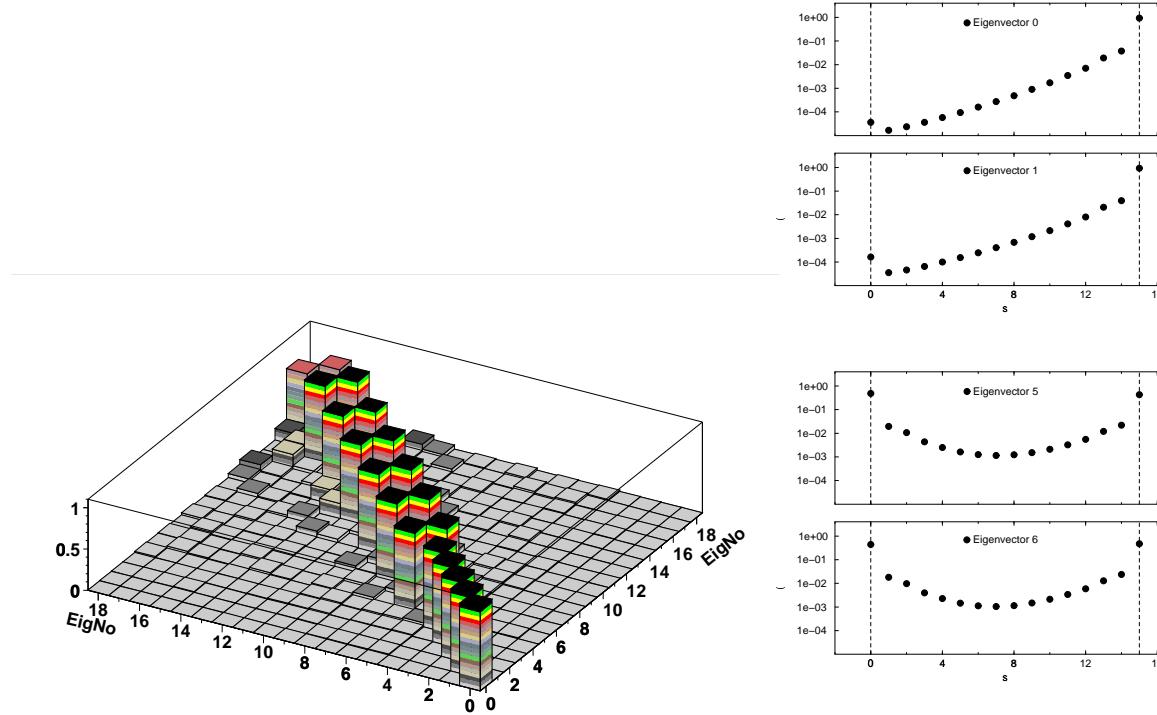
<sup>1</sup>D.B. Kaplan, Phys. Lett. B288, 342 (1992), hep-lat/9206013.

<sup>2</sup>Y. Shamir, Nucl. Phys. B406, 90 (1993), hep-lat/9303005; V. Furman and Y. Shamir, Nucl. Phys. B439, 54 (1995), hep-lat/9405004; and references cited therein.

RIKEN-BNL-Columbia-KEK (hep-lat/0007038): DWF works well,

- fermion near-zero modes are well understood, through

- 4-norm of the eigenvectors of  $D_H = \gamma_5 R_5 D$ ,
- matrix elements of  $(\Gamma_5)_{ss'} = \delta_{ss'} \text{sign} \left( \frac{L_s - 1}{2} - s \right)$  among them:



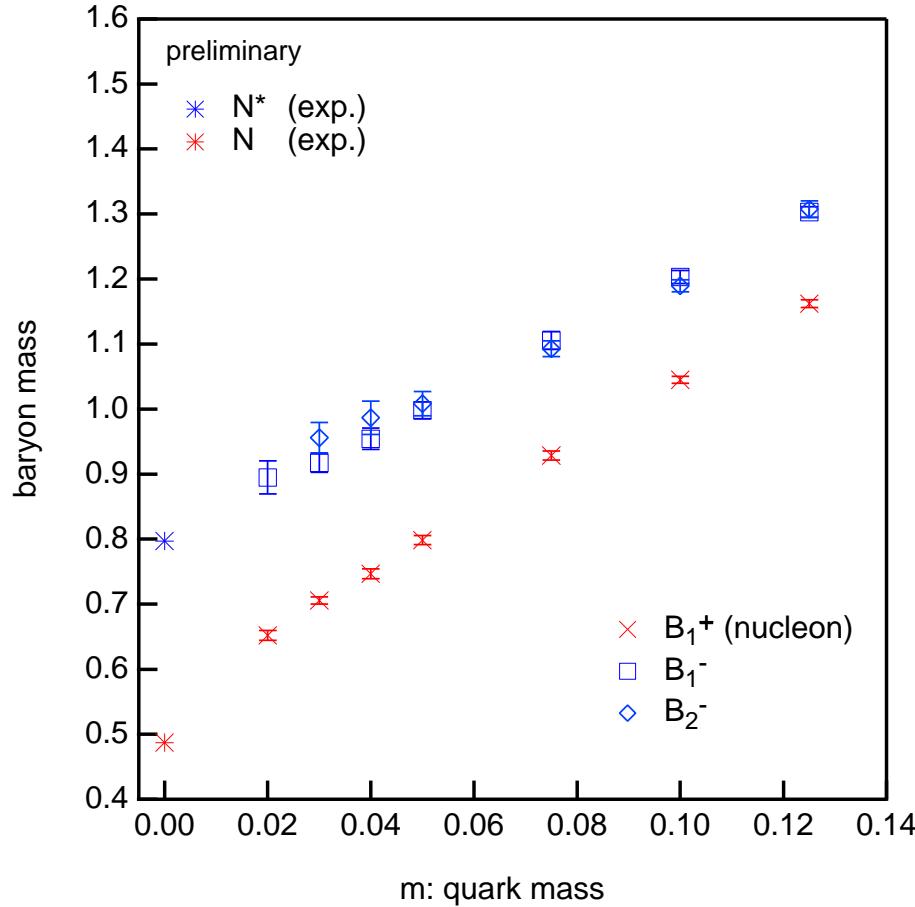
- Small induced chiral symmetry breaking described by  $m_{\text{res}}$ ,
  - $m_{\text{res}}$  decreases as  $\beta$  or  $L_s$  increases,
  - $m_{\text{res}}/m_{\text{strange}} = 0.033(3)$  at  $\beta = 6.0$  and  $L_s = 16$ .
- NPR works well with nearly continuum symmetries<sup>3</sup>.

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<sup>3</sup>C. Dawson's talk and forthcoming publications from RBCK QCD Project.

$T = 1/2$  mass spectrum:  $N(939)$  and  $N'(1440)$  with positive parity, and  $N^*(1535)$  with negative parity

- NR quark models and bag models fail here.
- Quenched DWF works well for  $N(939)$ - $N^*(1535)$  parity partner mass splitting <sup>4</sup>:



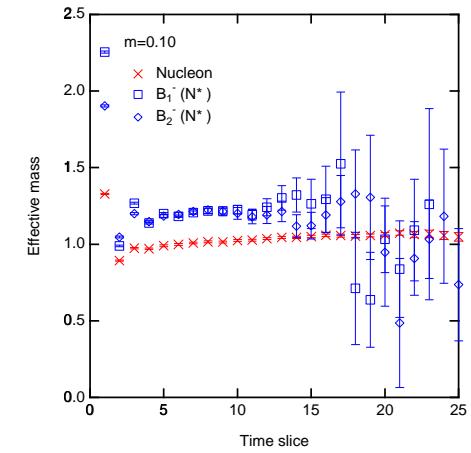
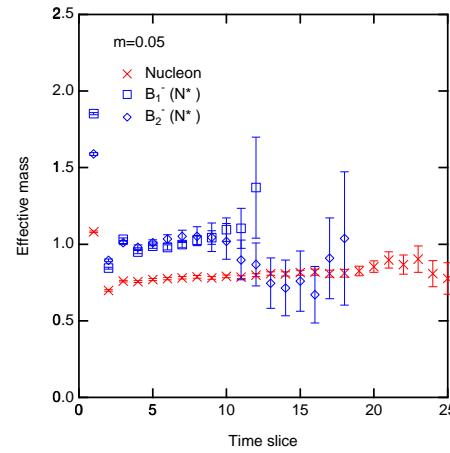
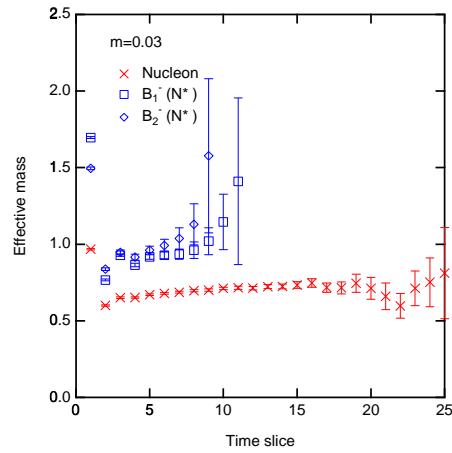
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<sup>4</sup>See S. Sasaki, hep-lat/0004252 for more detail.

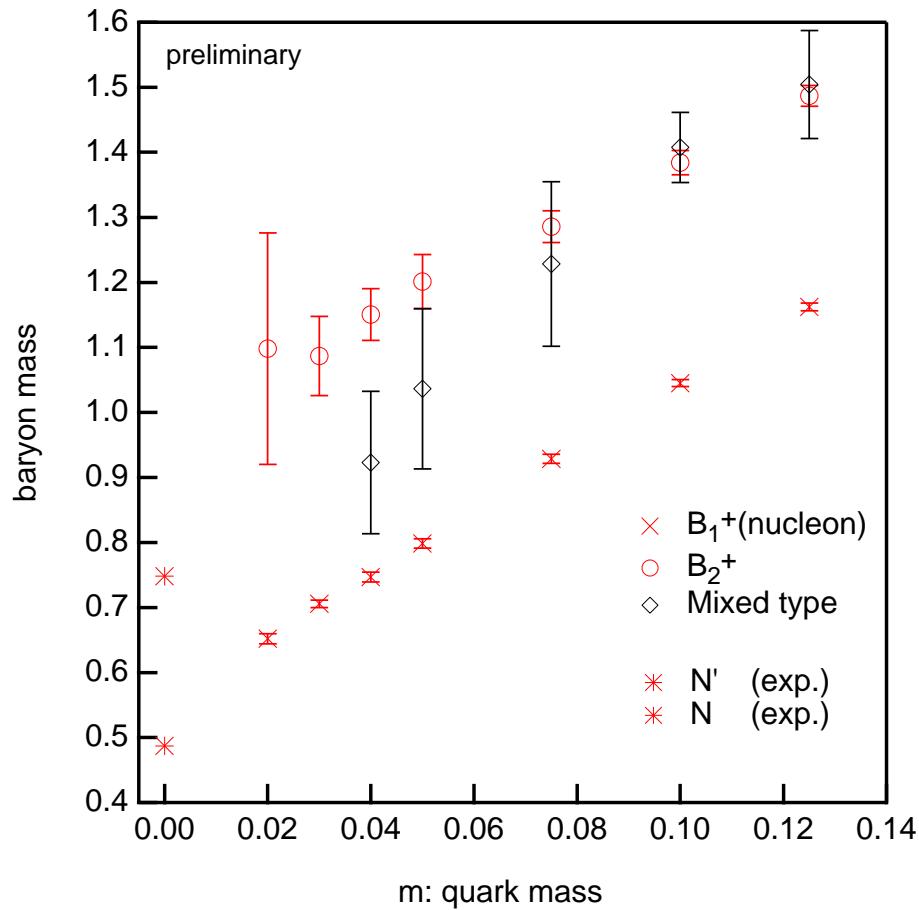
- Effective mass from baryon operators

- $J^P = (1/2)^+$ :  $B_1^+ = \epsilon_{abc}(u_a^T C \gamma_5 d_b) u_c$  and  $B_2^+ = \epsilon_{abc}(u_a^T C d_b) \gamma_5 u_c$
- $J^P = (1/2)^-$ :  $B_1^- = \gamma_5 B_1^+ = \epsilon_{abc}(u_a^T C \gamma_5 d_b) \gamma_5 u_c$  and  $B_2^- = \gamma_5 B_2^+ = \epsilon_{abc}(u_a^T C d_b) u_c$

with appropriate boundary conditions in time to reduce backward propagating contamination.



- Quenched DWF may even work for  $N'(1440)$ :



So DWF seems to have a better prospect for nucleon matrix elements:

$G_A$  is a particularly interesting exercise,

- particularly easy with DWF:  $Z_A = Z_V$  can be and is maintained.
- interesting to see how well quenched calculation works for a well-known example of soft-pion,
  - Goldberger-Treiman relation:  $G_A/G_V \simeq f_\pi g_{\pi N}/m_N \simeq 1.31$ ,

History

- NR quark model gives 5/3,
- MIT bag model gives 1.07,
- Lattice calculations typically underestimates by 20 %:

type	group	source	lattice	$\beta$	configs	$G_A$
quenched	KEK <sup>a</sup>	local	$16^3 \times 20$	5.7	260	0.985(25)
	Liu et al <sup>b</sup>	local	$16^3 \times 24$	6.0	24	1.18(11)
		point-split				1.20(10)
	DESY <sup>c</sup>	point-split	$16^3 \times 32$	6.0	1000	1.07(9)
full( $N_f = 2$ )	SESAM <sup>d</sup>	local	$16^3 \times 32$	5.6	200	0.907(20)

<sup>a</sup>M. Fukugiga, Y. Kuramashi, M. Okawa and A. Ukawa, Phys. Rev. Lett. 75, 2092 (1995).

<sup>b</sup>S.J. Dong, J.-F. Lagaë and K.F. Liu, Phys. Rev. Lett. 75, 2096 (1995); K.F. Liu, S.J. Dong, T. Draper and J.M. Wu, Phys. Rev. D49, 4755 (1994).

<sup>c</sup>M. Göckeler et al, Phys. Rev. D53, 2317 (1996); S. Capitani et al, in Proc. Lattice 98, Nucl. Phys. B (Proc. Suppl.) 73, 294 (1999); S. Capitani et al, Proc. DIS 99, Nucl. Phys. B (Proc. Suppl.) 79, 548 (1999) in which a value 1.14(3) is quoted.

<sup>d</sup>S. Güsken et al, Phys. Rev. D59, 114502 (1999)

- all with Wilson fermions,
- so  $Z_A \neq Z_V$  and other renormalization complications.

Our formulation follows the standard one,

- Two-point function:  $G_N(t) = \text{Tr}[(1 + \gamma_t) \sum_{\vec{x}} \langle TB_1(x)B_1(0) \rangle]$ , using  $B_1 = \epsilon_{abc}(u_a^T C \gamma_5 d_b) u_c$  for proton,
- Three-point functions,

– vector:  $G_V^{u,d}(t, t') = \text{Tr}[(1 + \gamma_t) \gamma_5 \sum_{\vec{x}'} \sum_{\vec{x}} \langle TB_1(x') V_t^{u,d}(x) B_1(0) \rangle]$ ,

– axial:  $G_A^{u,d}(t, t') = \frac{1}{3} \sum_{i=x,y,z} \text{Tr}[(1 + \gamma_t) \gamma_i \gamma_5 \sum_{\vec{x}'} \sum_{\vec{x}} \langle TB_1(x') A_i^{u,d}(x) B_1(0) \rangle]$ .

with fixed  $t' = t_{\text{source}} - t_{\text{sink}}$  and  $t < t'$ .

- From the lattice estimate

$$g_{\Gamma}^{\text{lattice}} = \frac{G_{\Gamma}^u(t, t') - G_{\Gamma}^d(t, t')}{G_N(t)},$$

with  $\Gamma = V$  or  $A$ , the continuum value

$$G_{\Gamma} = Z_{\Gamma} g_{\Gamma}^{\text{lattice}},$$

is obtained.

- Non-perturbative renormalizations, defined by

$$[\bar{u}\Gamma d]_{\text{ren}} = Z_{\Gamma} [\bar{u}\Gamma d]_0,$$

satisfies  $Z_A = Z_V$  well, so that

$$\left( \frac{G_A}{G_V} \right)^{\text{continuum}} = \left( \frac{G_A^u(t, t') - G_A^d(t, t')}{G_V^u(t, t') - G_V^d(t, t')} \right)^{\text{lattice}}.$$

$G_A$  is also described as  $\Delta u - \Delta d$ :

- $\Delta q$  is defined by  $\langle p, s | \bar{q} \gamma_5 \gamma_\mu q | p, s \rangle = 2s_\mu \Delta q$ ,
- with  $s$  satisfying  $sp = 0$  and  $s^2 = -1$ ,
- related to longitudinal parton distribution,  $\int_0^1 dx [q_\uparrow(x) - q_\downarrow(x)]$ .

Tensor charge is defined in a similar manner, with  $\sigma_{\mu\nu} = [\gamma_\mu, \gamma_\nu]/2$ :

- $\delta q$  is defined by  $\langle p, s | i \bar{q} \sigma_{\mu\nu} \gamma_5 q | p, s \rangle = 2(s_\mu p_\nu - s_\nu p_\mu) \delta q$ ,
- related to transverse parton distribution,  $\int_0^1 dx [q_\perp(x) - q_\top(x)]$ ,
- on the lattice, define  $G_T^q(t, t')$  by inserting  $T_i^q = \bar{q} \gamma_t \gamma_i \gamma_5 q$  at  $t'$ , and

$$\delta q^{\text{lattice}} = \frac{G_T^u(t, t') + G_T^d(t, t')}{G_N(t)}$$

is obtained,

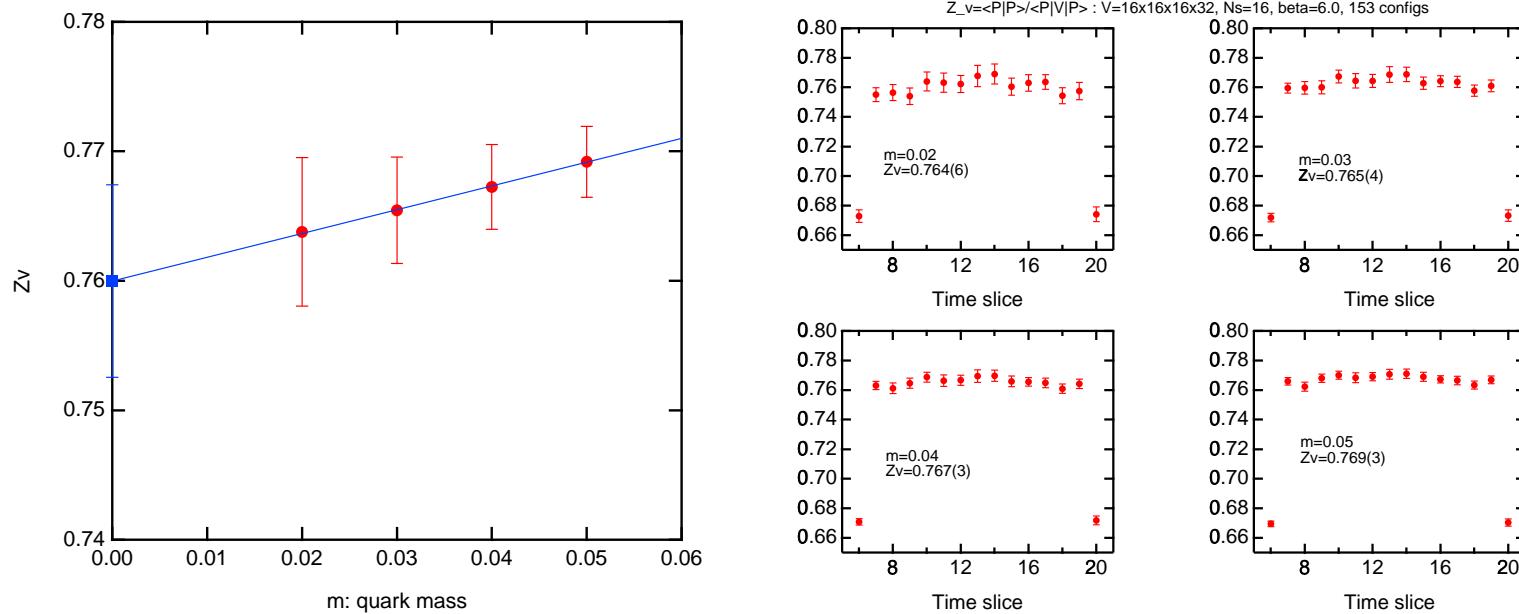
- need  $Z_T$ ,
- scheme/scale dependent.

In the heavy quark limit:  $\Delta u = \delta u = 4/3$  and  $\Delta d = \delta d = -1/3$ .

## Numerical calculations

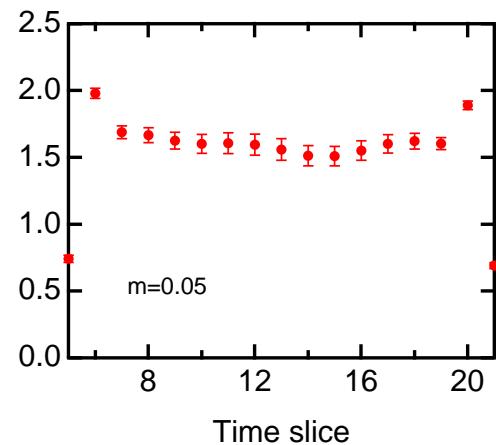
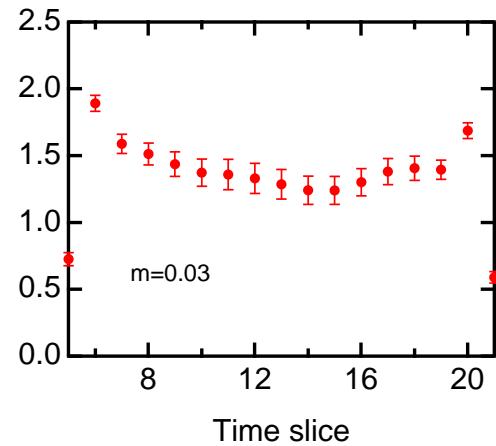
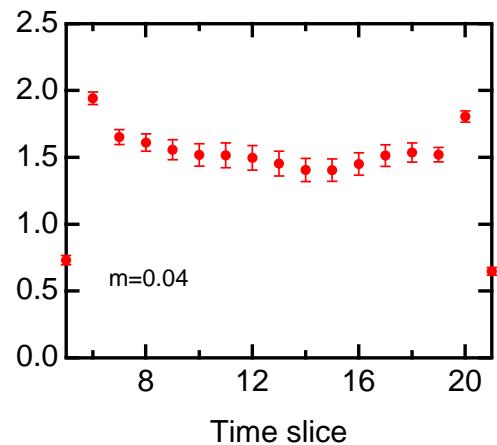
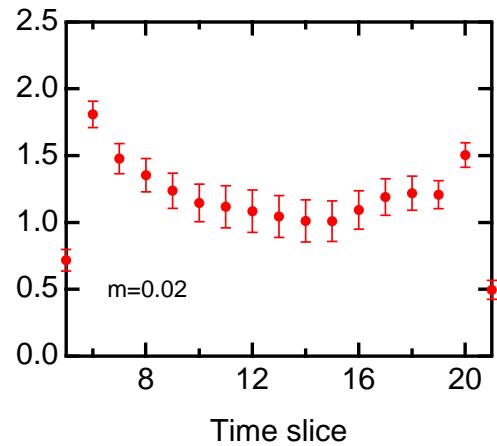
- RIKEN-BNL-Columbia-KEK QCDSF,
- 150-200 gauge configurations, using a heat-bath algorithm,
- $\beta = 6.0$ ,  $16^3 \times 32 \times 16$ ,  $M_5 = 1.8$ ,
- source at  $t = 5$ , sink at 21, current insertions in between.

$Z_v = 1/G_v^{\text{lattice}}$  is well-behaved,

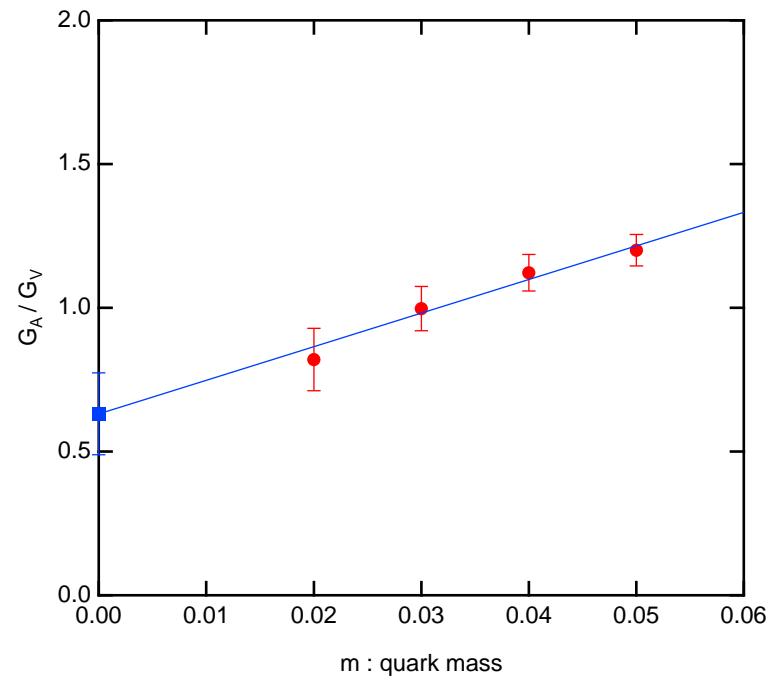
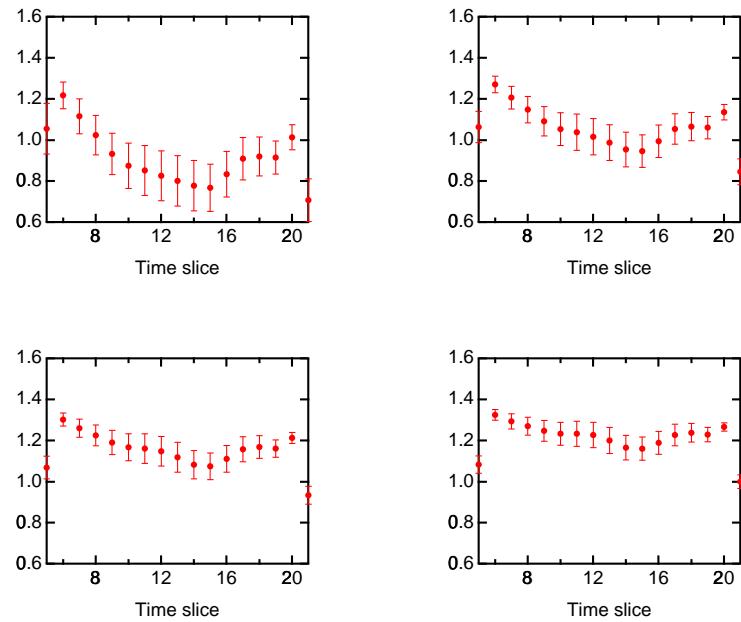


- the value  $0.764(6)$  at  $m_f = 0.02$  agrees well with  $Z_A = 0.7555(3)$  from  
 $- \langle A_\mu^{\text{conserved}}(t) \bar{q} \gamma_5 q(0) \rangle = Z_A \langle A_\mu^{\text{local}}(t) \bar{q} \gamma_5 q(0) \rangle$  (RBCK hep-lat/0007038),
- linear fit gives  $Z_v = 0.760(7)$  at  $m_f = 0$ .

$G_A^{\text{lattice}}$ : plateaux are seen in  $10 \leq t \leq 16$ ,

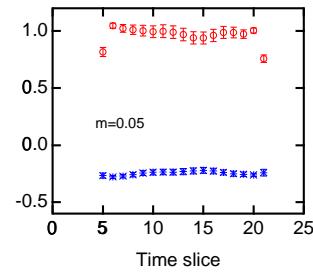
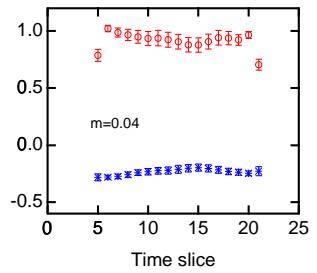
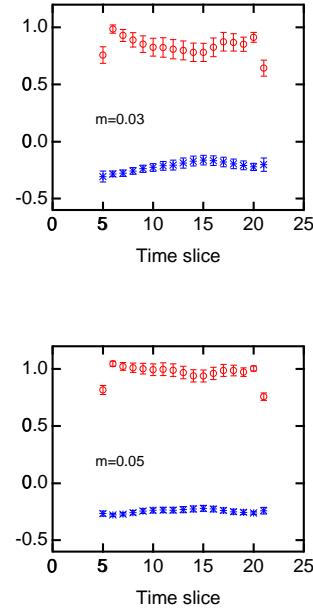
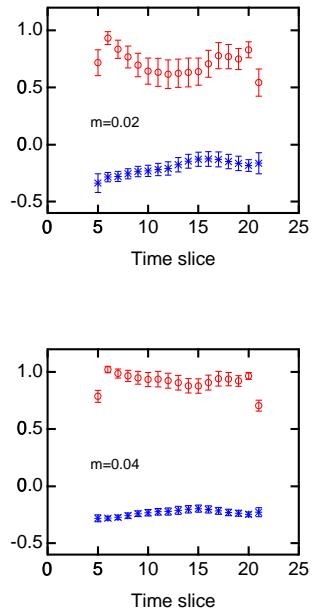
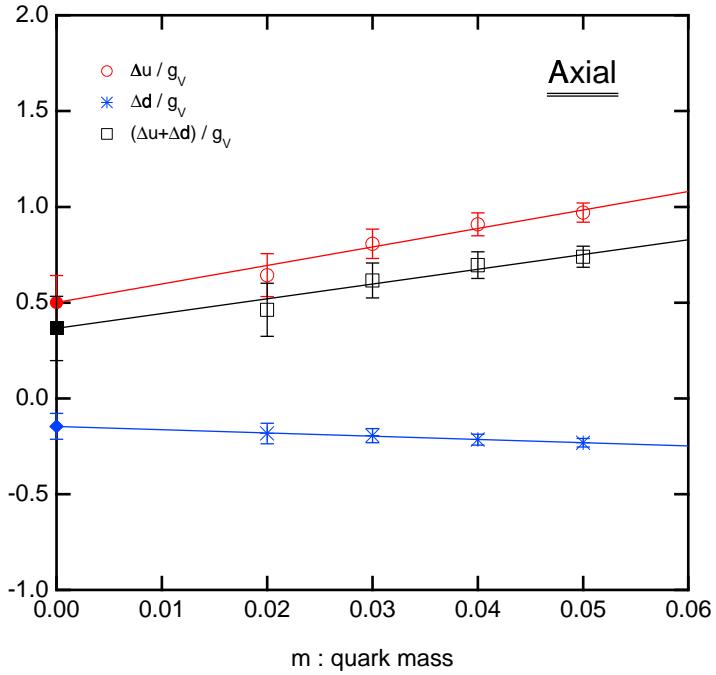


$G_A/G_V$ : averaged in  $10 \leq t \leq 16$ ,



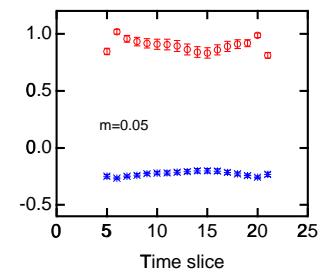
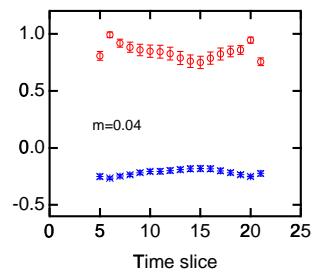
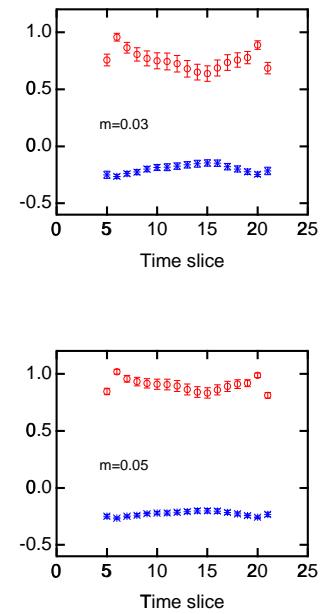
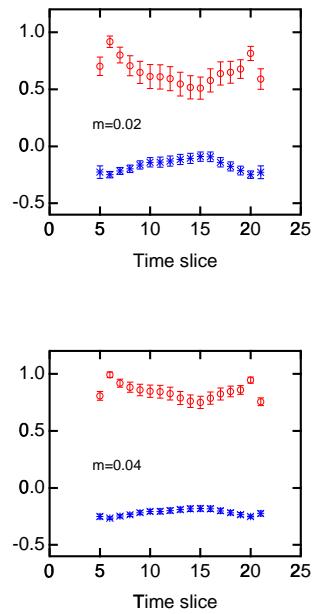
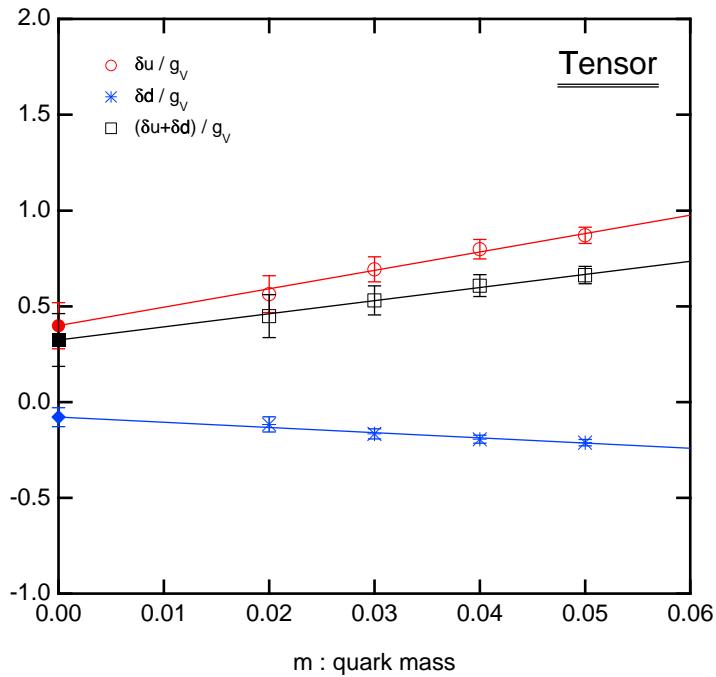
- linear extrapolation yields  $0.63(14)$  at  $m_f = 0$ .

$\Delta q$



- $\Delta u / G_v = 0.50(14)$  and  $\Delta d / G_v = -0.15(7)$  by linear extrapolation to  $m_f = 0$ .

$\delta q$



- $\delta u / G_v = 0.40(12)$  and  $\delta d / G_v = -0.08(5)$  by linear extrapolation to  $m_f = 0$ ,
- a preliminary value for  $Z_T / Z_A$  is  $1.1(1)$ <sup>5</sup>.

<sup>5</sup>A forthcoming RBCK paper.

Summary:

- Relevant three-point functions are well behaved in DWF,
  - $Z_V = Z_A$  is well satisfied, 0.760(7) and 0.7555(3),
  - linear extrapolation to  $m_f = 0$  gives
    - $G_A/G_V = 0.63(14)$ ,
    - $\Delta q/G_V = 0.37(17)$ ,
    - $(\delta q/G_V)^{\text{lattice}} = 0.32(14)$ , with a preliminary  $Z_T \sim 1.1(1)Z_A$ , in progress.
  - Further study required to check systematic errors arising from
    - finite lattice volume,
    - excited states (small separation between  $t_{\text{source}}$  and  $t_{\text{sink}}$ ),
    - quenching (zero modes, absent pion cloud, ...),
- especially in the lighter quark mass region.